SECTION – A

(Multiple Choice Questions)

Each question carries 1 mark

1. Which of these relations on set A where $A = \{1, 2, 3, 4\}$ are equivalence relation?

(a) $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

(b) $R_2 = \{(1, 4), (2, 2), (3, 3), (4, 1), (4, 2), (4, 4)\}$

(c) $R_3 = \{(1, 1), (1, 2), (1, 3)\}$

(d) $R_4 = \{(1, 1), (1, 2), (1, 4), (2, 2), (2, 4), (3, 3), (4, 1), (4, 4)\}$

2. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

3. The principal value of $\text{cosec}^{-1}(2)$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{5}$

4. The order of the matrix $\begin{bmatrix} a & 1 & x \\ a^2 - b & \sqrt{3} & 2 \\ \frac{-2}{5} & 5 & 0 \end{bmatrix}$ is

(a) 3×3 (b) 2×2 (c) 3×2 (d) 2×3

5. Maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix}$, θ is a real number is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

6. The function f, defined by $f(x) = \begin{cases} kx^2, & \text{if } x \geq 1 \\ 4, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, then $k =$

(a) 4 (b) –4 (c) 2 (d) –3

7. Function f, defined by $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is

(a) continuous at $x = 0$ (b) not continuous at $x = 0$

(c) differentiable at $x = 0$ (d) none of these

8. If $x = at^2, y = 2at$, then $\dfrac{d^2y}{dx^2}$ is

(a) $\dfrac{1}{t}$ (b) $-\dfrac{1}{t^2}$ (c) at^2 (d) $\dfrac{-1}{2at^3}$

9. $\int_0^{\frac{\pi}{2}} \dfrac{dx}{1+\sin x}$ equals to

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{3}{2}$

10. The value of integral $\int_0^{\frac{\pi}{4}} \dfrac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ is

(a) log 2 (b) $\dfrac{1}{20}\log 2$ (c) $\dfrac{1}{20}\log 3$ (d) log 5

11. The value of integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x . \log\left(\dfrac{1+x}{1-x}\right) dx$ is

(a) 0 (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{2}$ (d) none of these

12. The area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ is

(a) $\dfrac{25}{6}$ sq units (b) $\dfrac{23}{6}$ sq units (c) $\dfrac{27}{8}$ sq units (d) $\dfrac{29}{6}$ sq units

P.T.O.

13. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(*a*) π (*b*) $\frac{\pi}{2}$ (*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{4}$

14. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ is equal to

(*a*) $\cos\sqrt{x} + C$ (*b*) $2\cos\sqrt{x} + C$ (*c*) $-2\cos\sqrt{x} + C$ (*d*) $\sqrt{x}\cos\sqrt{x} + C$

15. The degree of the differential equation representing the family of curves

$(x - a)^2 + y^2 = 16$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

16. The solution of the equation $(2y - 1)dx - (2x + 3)dy = 0$ is

(*a*) $\frac{2x-1}{2y+3} = C$ (*b*) $\frac{2y+1}{2x-3} = C$ (*c*) $\frac{2x+3}{2y-1} = C$ (*d*) $\frac{2x-1}{2y-1} = C$

17. If two events are independent, then

(*a*) they must be mutually exclusive

(*b*) the sum of their probabilities must be equal to 1

(*c*) (*a*) and (*b*) both are correct

(*d*) None of the above is correct

18. Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Then $P(A \mid B).P(A' \mid B)$ is equal to

(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{8}$ (*c*) $\frac{3}{20}$ (*d*) $\frac{6}{25}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. **Assertion (A):** The minor of 5 in the determinant $\begin{vmatrix} 2 & 5 & 8 \\ 0 & 3 & 7 \\ -1 & -2 & 6 \end{vmatrix}$ is -7.

    Reason (R): The determinant that is left by cancelling the row and column intersecting at a particular element is called the minor.

20. **Assertion (A):** If $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two adjacent sides of a parallelogram the area of parallelogram is $3\sqrt{14}$ square units.

    Reason (R): $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

SECTION – B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21. Which is greater $\tan 1$ or $\tan^{-1} 1$?

22. Find the non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 12 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 44 \\ 10 & 6x \end{bmatrix}$$

OR

If $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find $A' - B'$.

P.T.O.

23. Find the general solution of the differential equation $(x + y)dy + (x - y)\, dx = 0$.

24. If for three non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

25. A coin is tossed thrice and all eight outcomes are assumed equally likely. Let the event E be "the first throw results in head" and event F be "the last throw results in tail". Find whether events E and F are independent.

SECTION – C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Let T be set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$ show that R is an equivalence relation.

27. Find the local maxima and local minima, if any of the function f, given by

$$f(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}.$$

28. Determine the constant a and b, such that the function $f(x) = \begin{cases} ax^2 + 2b & , \text{ if } x > 2 \\ 2 & , \text{ if } x = 2 \\ 2ax - 2b & , \text{ if } x < 2 \end{cases}$ is continuous.

29. If $a \cos y = x \cos(a + y)$ with $\cos a \neq \pm 1$ prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{a \sin a}$.

OR

If $y = \sin(\sin x)$ prove that $\dfrac{d^2y}{dx^2} + \tan x \dfrac{dy}{dx} + y \cos^2 x = 0$

30. Find $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}\,dx$

OR

Find $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx$

31. Find $\int \frac{2}{(1-x)(1+x^2)}\,dx$

OR

Evaluate $\int \frac{dx}{1+8\sin^2 x}$

SECTION – D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. In the determinant $\begin{vmatrix} 1 & 3 & -4 \\ 1 & 0 & 6 \\ 2 & 1 & 4 \end{vmatrix}$, find the cofactor of each element and hence evaluate

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x + 3z = -9,\ -x + 2y - 2z = -4,\ 2x - 3y + 4z = +3$

33. Find the shortest distance between the two lines:

$\vec{r} = (2+\lambda)\hat{i} + (12-\lambda)\hat{j} + (\lambda+1)\hat{k}$

$\vec{r} = (3\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

OR

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 3\hat{j} - 15\hat{k}$ and $\vec{c} = \lambda\hat{i} + 3\hat{j} + 13\hat{k}$ is equal to one. Find the value of λ and hence find unit vector along $\vec{b} + \vec{c}$.

34. Solve the LPP graphically

Maximise $Z = 15x + 2y$, subject to constraints:

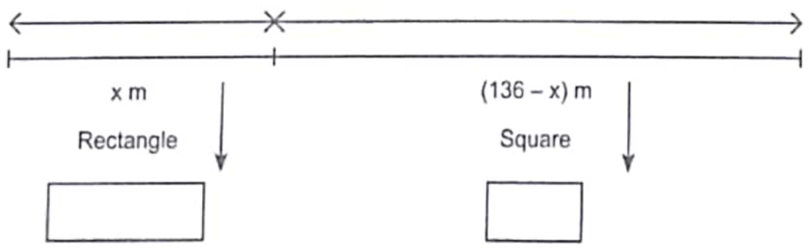
$x - 2y \le 2,\ 3x + 2y \le 12,\ -3x + 2y \le 3,\ x \ge 0,\ y \ge 0$

35. Find the particular solution of the differential equation $2x^2\dfrac{dy}{dx} - 2xy + y^2 = 0; y(e) = e$

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. A wire of length 136 m is to be cut into two pieces. One of the pieces is to be made into square and another into a rectangle as shown.



The length of rectangle is twice its breadth.

Answer the questions based on above information.

(i) Find the area of rectangle.

(ii) Find the area of square.

(iii) What length of wire is used to make rectangle, so that combined area of square and rectangle is minimum?

OR

(iii) Find the length of the side of square.

37. Two students Vivek and Mohan appears in an examination. Assume that the two events "Vivek passes" and "Mohan passes" are independent. The probability of Vivek passing an examination is $\frac{3}{5}$ and Mohan is $\frac{4}{5}$.

Answer the questions based on above information.

(i) What is the probability that both Vivek and Mohan pass the examination?

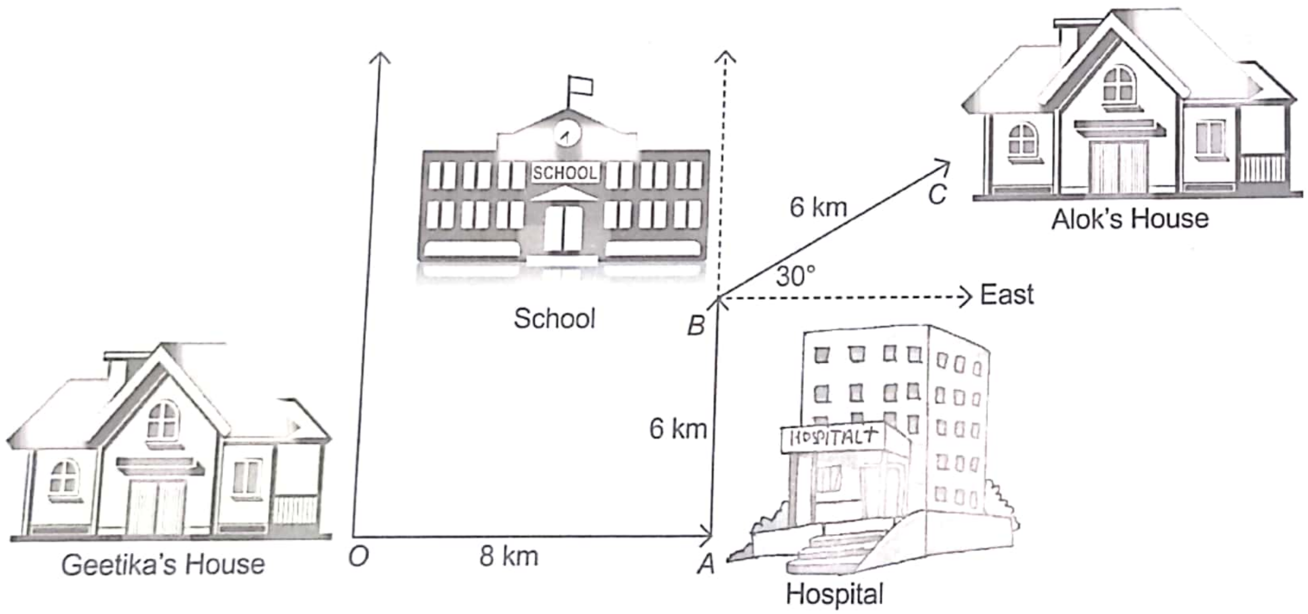
(ii) What is the probability that only Vivek passes the examination?

(iii) What is the probability that only Mohan passes the examination?

OR

(iii) Find the probability that only one of them pass the examination.

38. Geetika house is situated at Shalimar Bagh at point O, for going to Alok's house she first travels 8 km by bus in the East. Here at point A, a hospital is situated. From Hospital, Geetika takes an auto and goes 6 km in the North, here at point B school is situated. From school, she travels by bus to reach Alok's house which is at 30° East, 6 km from point B.



Based on the above information, answer the following questions.

(i) What is the vector distance between Geetika's house and school?

(ii) How much distance Geetika travels to reach school?